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“Telecom Spectrum Pricing Strategies”

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INTRODUCTION

The Indian economy and the sectors which were opened after liberalisation witnessed consistent growth in the Indian economy. The telecom sector post 1991-1992, has undergone major changes in terms of growth, diversity, technology and associated regulations.

The telecom service provider is one of the major verticals in telecom industry. The entire business model and survival of these service providers is based upon the license and spectrum issued by the Government. Thus license and spectrum are basic and main assets for telecom service providers. The Government has recently collected more than Rs. 1.06 laccrores from telecom service providers during 3G spectrum auction and BWA spectrum. The amount of revenue so collected is huge and thus contributed to partially overcome the issue of fiscal deficit. The UK, German governments also collected huge amount during such auctions in their respective countries.

The Global market is kept under speculation for the pricing of spectrum which is regulated differently in different geographies. The intensifying tariff war among telecommunication players along with a few declining financial parameters such as falling ARPU, declining profits, uncertain cashflows and BEP is likely to result in consolidation. In case of mergers, there are DOT regulations regarding spectrum.

As of today, spectrum is linked or bundled with licenses so the spectrum trading is not possible. Therefore in order to achieve an effective utilization of the spectrum, its pricing is also important.

ECONOMIC THEORY OF PRICING

The Price of anything is the rate at which it can be exchanged for anything else. Price is a central issue both for marketing and economics. In our paper we are focussing on the economic theory of pricing and not the marketing theory of pricing. There are some common and uncommon thoughts about pricing in Economics and Marketing.
The task of the economic theory is to explain why some goods are expensive and others are cheap. The commodities have price because they are useful and are scarce in relation to the use which people want to put them. Spectrum is very scarce and useful, so it bears economic price.

Transaction on the basis of a price may be considered from three distinct sides: those of the buyer, the seller and the wider industry or economy as a whole. In case of telecom industry, the telecom operators are buyers so their responses to prices assume that buyers behave rationally. It is further assumed that buyers always act so as to maximise their utility under certain constraints. On the basis of this theory, it is possible to logically derive the “law” of demand and also concepts like price elasticity.

From a seller’s perspective, during allocation of spectrum the seller would be Government where as post allocation operators would be the sellers. In the initial stage, Government being auctioning the spectrum has an objective of maximising the revenue and use it to minimise the budget deficit. However post allocation the sellers would be operators whose objective is to maximise the profit and wealth in long run. In this scenario, price determination is also analysed under different forms of competition.

The industry and economic-wide significance of prices has been traditionally of great importance to economics, there are two approaches, general equilibrium analysis and macroeconomics which are also considered in this paper.

**SPECTRUM AND ITS CHARACTERISTICS**
Spectrum is a range of electromagnetic radio frequencies used for transmission of voice, data and images. Mobile telecom operators send and receive frequencies to enable communication between two phones. Assignment of spectrum is governed by the national frequency allocation plan (NFAP) and the International Radio Regulations of the International Telecommunications Union (ITU). The Wireless Planning and Coordination (WPC) wing of the department of telecom (DoT) performs spectrum management functions in India, while the Standing Advisory Committee on frequency allocation formulates and reviews the NFAP and evolves technical criteria equipment standards. The specific characteristics are as follows;

Spectrum is a very scarce resource;

It is fully renewable;

Interdependencies due to signal overlay could cause interference if not managed properly;
All frequency bands are not alike. Different frequencies have different propagation characteristics and their usage varies accordingly.

Due to its unique nature pricing plays an important role.

**EXISTING CONDITION OF AUCTION**
The paper is also going to cover different methods of assigning the spectrum. During the initial allocation of spectrum by Government, Auctioning and Beauty contest are the most popular modes of spectrum. These two methods are used across the globe. The paper attempts to analyse the comparative features of both the methods. Auctions represent a form of assignment mechanism where the applicants determine the value to be charged.

Each method has its own pros and cons which are going to be covered in the paper. The process of 3G spectrum auction came into highlight when over $100 billion were gathered through 3G spectrum in Europe. This resulted in India also going for the auctioning of the 3G spectrum in 2010 and gaining over $15 billion.

The paper will also focus on the scenario in various countries where the need was felt to allow spectrum trading for various reasons after the initial allocation of the spectrum. Countries like New Zealand, Australia, UK, Germany, etc have liberalised their spectrum related policies. We will cover the steps taken by various governments to allow spectrum trading and how things have been after the trading was allowed.

**OBJECTIVE**
- Identifying parameters influencing the pricing model
- Develop pricing matrix
- Economic and commercial implication of various models

**DEFINING MODELS**
**NOTE** :- (numerical values assumed here are arbitrary; this is a tool wherein Telcos are expected to give in their input which will help to suggest them the exact type of Spectrum pricing model they should go forward with and the kind of model telcos should adopt in order to make maximum benefit out of their most valuable asset that is the spectrum)

Identifying the base parameters: the main parts that were made was
1. Economic: both Demand and supply are considered here

2. Commercial: here the two critical parameters were the cost and revenue

Again they are subdivided into many smaller parameters

**COMMERCIAL PARAMETERS**

Commercial parameters revolve around the Revenue and the cost as these are the two main components of a commercial viability of any business, as we want to use the spectrum as a entity that will generate revenue for the telcos therefore we have classified mainly 3 cost components

1. Sunk Cost

This might include costs of marketing a service or infrastructure costs that may have to be written off if a service fails.

2. Transaction cost:

Transaction costs are one-off costs at the point of sale and will be incurred by buyers and sellers. Penalties for breaking contracts associated with the existing services (e.g. site rental contracts). Both buyers and sellers will incur the costs of management time and legal support for negotiating a sale. It is also well known that firms review opportunities for change periodically not continuously because of the limited availability of management time to make strategic decisions (leading to selective intervention) and because of the scale of transaction costs. Sellers (and possibly buyers) will also incur costs associated with vacating the spectrum they sell if it is currently used and/or with the potential loss of option value if the spectrum is not used. The costs incurred by the seller associated with vacating spectrum and potentially moving to another band or another service could comprise one or more of:

- Write-off of the accounting cost of existing equipment if there is no second hand market.
- The costs of removing existing equipment and installing new equipment.
- The risk or actuality of revenue loss in the transition to the new arrangements.
- The costs of operating duplicate systems so as to maintain business continuity. This may be particularly acute for services with many end users and/or where the applications in question are safety critical.

The nature and scale of these costs will depend on the situation being considered and the risks faced by organisations when changing their spectrum use. However we note that these costs are often mentioned by incumbents when asked by the regulator to change their spectrum use and may in some circumstances explain low trading volumes by commercial organisations.

3. Opportunity Cost:
This is the cost associated with forgoing of the other alternative that we have. For example if Telco does spectrum trading then the cost associated with forgoing the opportunity of taking it directly from government.

<table>
<thead>
<tr>
<th>Entity</th>
<th>INPUT (percentage in decimal)</th>
<th>Weightage</th>
<th>Final value</th>
</tr>
</thead>
<tbody>
<tr>
<td>REVENUE (values for growth percentage)</td>
<td>Net present Value</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ROI</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Profit</td>
<td>0.4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>additional revenues</td>
<td>0.1</td>
<td>4</td>
</tr>
<tr>
<td>COST (As a percentage of overall cost for a financial year)</td>
<td>Infrastructure cost</td>
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<td>1</td>
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<tr>
<td></td>
<td>Marketing cost</td>
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<td>2</td>
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<tr>
<td></td>
<td>License cost (Recurring)</td>
<td>0.4</td>
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<td></td>
<td>Spectrum rental cost</td>
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<td>Transaction cost</td>
<td>Penalties for breaking contracts associated with the existing services (e.g. site rental contracts).</td>
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<td>1</td>
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<tr>
<td></td>
<td>Write-off of the accounting cost of existing equipment if there is no second hand market.</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>The costs of removing existing equipment and installing new equipment</td>
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<td>3</td>
</tr>
<tr>
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<td>Market Risk</td>
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### Technical Risk

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<th>Parameter</th>
<th>Value</th>
<th>Weight</th>
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<tbody>
<tr>
<td>The costs of operating duplicate systems so as to maintain business continuity.</td>
<td>0.3</td>
<td>2</td>
<td>0.6</td>
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<tr>
<td>SLA not fulfilled – forfeiting amount</td>
<td>0.4</td>
<td>3</td>
<td>1.2</td>
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</table>

### Opportunity Cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Weight</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price from spectrum allocation directly from government</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>Prices from bi-lateral trades</td>
<td>0.2</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>Auction prices</td>
<td>0.3</td>
<td>4</td>
<td>1.2</td>
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</table>

### Economic Parameters

<table>
<thead>
<tr>
<th>Entity</th>
<th>Parameters</th>
<th>Input (percentage in decimals)</th>
<th>Required Value</th>
<th>Weightage (1-4)</th>
<th>Final Value</th>
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</thead>
<tbody>
<tr>
<td>Demand</td>
<td>Forecast growth in ARPU</td>
<td>0.2</td>
<td>0.2</td>
<td>2</td>
<td>0.4</td>
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<tr>
<td></td>
<td>Income growth</td>
<td>1.2</td>
<td>1.2</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Increased Mau</td>
<td>0.3</td>
<td>0.3</td>
<td>3</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Unpenetrated Rural market</td>
<td>0.4</td>
<td>0.4</td>
<td>4</td>
<td>1.6</td>
</tr>
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<td></td>
<td>Unpenetrated Urban market</td>
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<td>0.2</td>
<td>1</td>
<td>0.2</td>
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<td></td>
<td>International roaming possibilities</td>
<td>0.3</td>
<td>0.3</td>
<td>2</td>
<td>0.6</td>
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<td></td>
<td>Competition from other operators and services</td>
<td>0.2</td>
<td>0.2</td>
<td>2</td>
<td>0.4</td>
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<td></td>
<td>Current</td>
<td>Economic</td>
<td>Supply</td>
<td>Economic</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>---------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td><strong>Current Utilization out of Maximum Capacity</strong></td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td><strong>Dropped calls as percentage of all call attempts</strong></td>
<td>0.01</td>
<td>0.99</td>
<td>3</td>
<td>2.97</td>
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<tr>
<td><strong>Latency</strong></td>
<td>0.2</td>
<td>0.8</td>
<td>4</td>
<td>3.2</td>
<td></td>
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<tr>
<td><strong>Number of Operators per circle out of total nationwide operators</strong></td>
<td>0.2</td>
<td>0.8</td>
<td>1</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td><strong>Unutilised Spectrum out of total spectrum</strong></td>
<td>0.3</td>
<td>0.3</td>
<td>2</td>
<td>0.6</td>
<td></td>
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<tr>
<td><strong>Spectral efficiency</strong></td>
<td>0.3</td>
<td>0.3</td>
<td>3</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td><strong>Final Output</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>1.0976923</strong></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th><strong>Commercial</strong></th>
<th><strong>Economic</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6722222222</td>
<td>1.097692308</td>
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</table>
ANALYSIS

Therefore from above it is obtained that

1. SPECTRUM AUCTION: The Indian regulation is extremely resistant towards this policy and it will take some time but the profitability and the economic importance of this model is extremely high, here what the operator is suppose to do is that if it has free spectrum then it will join hands with government and will go for a auction and if the demand high and the circle is lucrative then the bidding amount will be high which will ultimately benefit govt only

2. SPECTRUM TRADING: Again this will also face resistance from govt, but this is also very highly profitable commercial but economic importance is lesser than that of spectrum auction, Here in spectrum trading a part of spectrum can be sold to the other party who has the infrastructure and capital if the former doesn’t have effective utilization of the spectrum therefore its a Win Win situation for both the parties as well as govt as it will get higher tax out of this

3. SPECTRUM SHARING: It was recently given in NTP draft 2011, therefore the govt is flexible towards it, in this if one operator doesn’t have the spectrum at some location then it can share the spectrum of the lender based on SLA of sharing, therefore the spectrum utilization will be very high therefore may be its less commercial than that of spectrum trading but its economic importance is as high as spectrum auction

4. SPECTRUM POOLING: This was also proposed in NTP draft 2011 because its highly beneficial from spectrum utilization perspective though the commercial benefit and economic viability is less but still it stands a good business case
SPECTRUM AUCTION PRICING

TYPES OF AUCTION

In the English auction, users bid the highest price they are willing to pay for an item and the bidding activity stops when the pre-determined auction duration is complete. The item is sold to the highest bidder at their bid price. It allows dynamic adjustment of bidders’ valuations by giving information about other bidders. English auctions also allow the seller to specify a reserve price below which the item will not be sold.

In the Dutch auction, the auctioneer starts at a very high price – starting bid. The standing price is gradually lowered, typically by means of an exogenous counting device (a clock, or a pointer). The process continues until a bidder indicates a buy signal, i.e. raising his hand, at which time that bidder wins the unit. It raises sellers’ revenue if the bidder wants the item badly and it is considered to be fast. "Dutch auction" is also sometimes used to describe online auctions where several identical goods are sold simultaneously to an equal number of high bidders. In an Anglo-Dutch auction the auctioneer begins by running an ascending auction in which price is raised continuously until all but two bidders have dropped out. The two remaining bidders are then required to make a final sealed-bid offer that is not lower than the current asking price, and the winner pays his bid.

In the Sealed High-Bid Auction or First-Price Sealed-Bid Auction (FPSB, all bidders simultaneously submit bids and they do not know the bid of the other participants. The highest bidder pays the price they submitted. The multi-object form of the first price auction is called discriminatory auction. The format is largely used for procurement, also used for refinancing credit and foreign exchange.

The Vickrey auction or the second price auction operates similarly with the English auction. The highest bidder obtains the item at the price offered by the second highest bidder. The advantage is that the bidders bid what they think the item is worth and there is no influence on what the others will bid.

Comparing the different types of auctions we conclude as following:

- Bidders tend to underbid what they believe the item is truly worth in hopes of getting the item for less, or in order to avoid the winner’s curse (It is also called Bid Shading, a tendency for the winning bid in an auction to exceed the intrinsic value of the item purchased)

- All auctions are theoretically equivalent, but in practice Dutch auctions will produce less revenue than sealed first-price auctions

- The English and Vickrey auctions are equivalent in the outcome, under private values, but not strategically; in an English auction bidders can respond to rivals’ bids.

- Finally the English, Sealed Bid and Classic Dutch are all pay-your-bid auctions except of the Vickrey that follows the “second price” rule.
PEAK LOAD PRICING

Peak-load pricing (PLP) refers to the pricing of economically non-storable commodities whose demand varies periodically. PLP is often used by electricity, telephone and other public utilities and also the Internet as a means of reflecting the investment they have made to meet peak demand for their services.

This pricing scheme corresponds to high competition and price discrimination problems when efficiency is needed due to the increasing role for services in the economy. Therefore it is ideal for industry practice and real world applications. In the rest of this section we will describe the PLP model and how its constraints could be translated into the spectrum pricing problem.

PLP refers to the “on-peak” and the “off-peak” time period. On peak usually is described when the demand of the product exceeds the supply and additional units should be produced, when off peak is considered as the condition when supply satisfies the demand. According to the literature the market conditions determine which period is “high” and which season is “low.”. Usually the model is applied to monopolistic markets where the units’ producer i.e. the regulated sector has full control of the management environment but today is spread over competitive industries i.e. airlines and hotels.

The constraints of the PLP after deriving efficient prices are mainly to maximize the Welfare profit, called also the net social benefit and optimize the producer’s profit in terms of revenues. As illustrated the producer charges a higher price (PHi),

- PHi=b+β during peak times (DHi) and
- a lower price (PLo), PLo=b, during off-peak times (DLo).

The b equals the operational cost and the β is the cost of providing a unit of additional capacity.
W=TR+S-TC (1), where

W=net social benefit
TR=total revenue
S=consumers’ surplus
TC=total costs

\[ W = \int_{0}^{X} P(x)dx - C(X), \quad (2) \]

P(X) = demand function
C(X) = total cost function
X= (x1,x2,...xn), is the total demand

After maximizing equation (2), the price equals the marginal cost. In other cases during the peak hour diverse technology might be deployed to help fulfill the demand. For example during the peak-
period it may be more economical to employ an additional technology type to meet the peak-period demand, anticipating lower construction costs and higher operating costs, thus offering cost advantages. A very interesting case is the PLP with demand and supply uncertainty. “Efficient pricing rules require consideration of willingness to pay for services rendered, when supply is sufficient to meet demand, and for services not rendered plus any rationing costs incurred in excess demand states”. In this case the possibility of “outage” arises, which is the excess demand in certain states. The costs associated with outage are separated into three elements

(1) rationing cost, which is the cost incurred by utility in allocated scarce supply

(2) disruption cost

(3) surplus loss.

Assuming multiple time periods, the price in each time period should be set equal to the expected, deterministic short run marginal cost, including the expected marginal disruption and rationing costs.

On the capacity side, per unit cost equals the expected marginal disruption and rationing costs. Summarizing the utility needs to use both price rationing as well as quantity rationing to efficiently allocate available capacity.

In a single pricing period with only one technology used the optimal price could be calculated as following. The optimal price will include also the willingness to pay (Λ). Assuming that our system is characterized with multiplicative uncertainty the optimal price will be the product of maximizing the Welfare function for single technology and single pricing period:

\[ P^{**} = b + \gamma \left( \frac{\beta}{\alpha} \right) - \Lambda \]

\[ P^{**} = \text{Optimal price} \]

Where b usually is the operating cost at the agent’s side to handle as many operators and bids he is configured i.e. Internet cost, software updates

\( \beta = \text{usually is the cost of producing an additional unit, in our case would be the opportunity cost if the licenses were assigned to different operators in that particular time window T} \)

\( \alpha = \text{equals the ratio of Firm Generation Capacity to Peak Load.} \)

\( \gamma = \text{is the load below which below which calls attempts cannot be catered} \)
BASICS OF SECONDARY MARKET PRICING

\[ U(q_1, q_2, I) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2} (\beta_1 q_1^2 + \beta_2 q_2^2 + 2\gamma q_1 q_2) + I \] .

For simplicity let us assume that \( \beta_1 = \beta_2 = 1 \). Thus, utility is quadratic in the consumption of \( q \)-goods and linear in the consumption of other goods, \( I \). The parameter \( \gamma \in [-1, 1] \) measures the substitutability between the products. If \( \gamma = 0 \), each firm has monopolistic market power, while if \( \gamma = 1 \), the products are perfect substitutes. A negative \( \gamma \) implies that the goods are complementary. Finally, \( \alpha \) measures quality in a vertical sense. Other things equal, an increase in \( \alpha \) increases the marginal utility of consuming good \( i \).

It is straightforward to generalize the utility function to allow for \( n \) firms producing one product variety each.

\[ U(q, I) = \sum_{i=1}^{n} q_i \alpha_i - \frac{1}{2} \sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i=1}^{n} q_i q_j + I \]

Consumers maximize utility subject to the budget constraint \( \sum p_i q_i + I \leq m \), where \( m \) denotes income and the price of the composite good is normalized to one. The first-order condition determining the optimal consumption of good \( k \) is
\[ \frac{\partial U}{\partial q_k} - \alpha_k - q_k - \gamma \sum_{j \neq k} q_j - p_k = 0 \]  

(1)

**Cournot Competition**

Firm \( k \)'s inverse demand function can be solved for directly from expression 1.

\[ p_k(q_k, q_j) = \alpha_k - q_k - \gamma \sum_{j \neq k} q_j \]

Firms set quantities to maximize profits, \( \pi_k \), taking the other firms’ quantities as given. If costs are normalized to zero, firm \( k \)'s reaction function equals

\[ q_k(q) = \frac{\alpha_k - \gamma \sum_{j \neq k} q_j}{2}. \]

Summing over all firms and noting that

\[ \sum_{i=1}^{n} q_i = q_k + \sum_{j \neq k} q_j \]
\[ \sum_{i=1}^{n} \alpha_i = \alpha_k + \sum_{j \neq k} \alpha_j \]

we can solve for demand and price in equilibrium

\[ q^c_k = p^c_k = \frac{\alpha_k [\gamma(n-2)+2] - \gamma \sum_{j \neq k} \alpha_j}{(2 - \gamma)[\gamma(n-1)+2]} \]

where superscript \( C \) indicates Cournot equilibrium. Thus, firm \( k \)'s equilibrium price and quantity depend on the average quality if its competitors products but are independent of the exact distribution of product qualities across firms.
SPECTRUM TRADING PRICING

The competitive pricing model will be applicable in case of spectrum trading, where it will be assumed that a primary service provider is aware of the existence of other primary service providers and all of the primary services compete with each other to achieve the highest individual profit. We assume that the competition here occurs in terms of spectrum pricing. That is, given the spectrum prices offered by other primary services, one primary service chooses the price for its own spectrum so that its individual profit is maximized.

SOLUTIONS OF SPECTRUM TRADING PRICING MODEL

A. Utility of Secondary Service

If the spectrum available to the secondary service creates high utility, the demand is high. To quantify the spectrum demand, we consider the utility gained by the secondary users. We adopt the following commonly used quadratic utility function

\[ U(b) = \sum_{i=1}^{N} b_i k_i^{(s)} - \frac{1}{2} \left( \sum_{i=1}^{N} b_i^2 + 2\nu \sum_{i \neq j} b_i b_j \right) - \sum_{i=1}^{N} p_i b_i \]

where \( b \) is a vector of shared spectrum sizes from all the primary services, i.e., \( b = [b_1 \ldots b_i \ldots b_n] \), \( p_i \) is the price offered by primary service \( i \) (\( N \) is the total number of primary services, i.e., spectrum sellers). Note that \( k_i^{(s)} \) denotes the spectral efficiency of wireless transmission by the secondary users using frequency spectrum \( F_i \) owned by primary service \( i \).

This utility function takes the spectrum substitutability into account through the parameter \( \nu \) (0.0 \( \leq \nu \leq 1.0 \)). That is, if a secondary user uses a multi-interface network adapter, it is able to switch among the frequency spectra freely depending on the offered price. This spectrum substitutability parameter \( \nu \) is defined as follows. When \( \nu = 0.0 \), a secondary user cannot switch among the
frequency spectra, while for $\nu$ 1.0 the secondary user can switch among the operating frequency spectra freely.

The demand function for spectrum $F_i$ at the secondary service can be obtained as following:

$$D_i(p) = \frac{(k_i^{(s)} - p_i)(\nu(N - 2) + 1) - \nu \sum_{i \neq j} (k_j^{(s)} - p_j)}{(1 - \nu)(\nu(N - 1) + 1)}$$

where $p$ denotes a vector of prices offered by all primary services in the market (i.e., $p=[p_1 \ldots p_i \ldots p_N]^T$). The demand function in (2) can be rewritten as follows: $D_i(p) = D_1(p_i) - D_2p_i$, where $p_i$ denotes the vector of prices of all primary services except service $i$, $D_1(p_i)$ and $D_2$ are constants for given $p_j$ for $i \neq j$ which are given as

$$D_1(p_{-i}) = \frac{k_i^{(s)}(\nu(N - 2) + 1) - \nu \sum_{i \neq j} (k_j^{(s)} - p_j)}{(1 - \nu)(\nu(N - 1) + 1)}$$

$$D_2 = \frac{\nu(N - 2) + 1}{(1 - \nu)(\nu(N - 1) + 1)}.$$

B. Revenue and Cost Functions for a Primary Service

For a primary service, there are two sources of revenue from primary users and secondary users. However, a cost is involved which is a function of QoS performance degradation of ongoing primary users due to sharing the radio spectrum with secondary service. We assume that the primary users are charged at a flat rate for a guaranteed amount of bandwidth. However, if the required bandwidth cannot be provided, a primary service offers “discount” to the primary users, and this is considered as the cost of sharing spectrum with the secondary service.

Let $R_i^p$ denote the revenue gained from primary users served by primary service $i$, $R_i^s$ denote the revenue gained from sharing spectrum with secondary users, and $C_i$ denote the cost due to QoS degradation of primary users. Then, the revenue and cost functions can be defined as follows:

$$R_i^p = p_i b_i,$$

$$R_i^s = c_1 M_i, \quad C_i(b_i) = c_2 M_i \left( B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i} \right)^2$$

where $b_i$ and $p_i$ denote, respectively, the spectrum size shared with secondary service and the corresponding price, and $c_1$ and $c_2$ denote constant weights for the revenue and cost functions at the primary service, respectively. Here, $B_i^{req}$ denotes bandwidth requirement per user, $W_i$ denotes spectrum size, $M_i$ denotes the number of ongoing primary users, and $k(p)_i$ denotes spectral efficiency of wireless transmission for primary service $i$. Note that revenue from the primary users is a linear function of the number of ongoing users, while revenue from secondary users is a linear function of the shared spectrum size given the spectrum price. The cost is proportional to the square of the difference between bandwidth requirement and allocated bandwidth to a primary user.
Solution of Competitive Pricing Model

We use a noncooperative game to model the price competition among primary services. The players (i.e., sellers in an oligopoly market) in this game are the primary services. The strategy of each of the players is the price per unit of spectrum. The payoff for each primary service $i$ (denoted by $P_i$) is the individual profit due to selling spectrum to the secondary service. Again, based on the demand, revenue, and cost functions, the individual profit of each primary service can be expressed as follows: $P_i(p) = R_i^w + R_i^l - C_i$, where $p$ denotes a vector of prices offered by all of the players (i.e., primary services) in the game.

We consider the Nash equilibrium [24] as a solution of this price competition. In this case, the Nash equilibrium is obtained by using the best response function which is the best strategy of one player given others’ strategies. The best response function of primary service $i$, given a vector of prices offered by other primary services $p_{-i}$, is defined as follows:

$$\mathcal{B}_i(p_{-i}) = \arg \max_{p_i} \mathcal{P}_i(p_i, p_{-i}).$$

The vector $P^* = [...................p^*_i................]$ denotes a Nash equilibrium (i.e., solution) of this game on competitive pricing for

$$p_i^* = \mathcal{B}_i(p_{-i}^*), \quad \forall i$$

where $p^*_j$ denotes the vector of best responses for player $j$ for $j \neq i$. Mathematically, to obtain the Nash equilibrium, we have to solve the following set of equations:

$$\frac{\partial \mathcal{P}_i(p)}{\partial p_i} = 0$$

for all $i$. In this case, the size of the shared bandwidth $b_i$ in the individual profit function is replaced with spectrum demand $D_i p$, and then the profit function can be expressed as follows:

$$\mathcal{P}_i(p) = p_i \mathcal{D}_i(p) + c_1 M_i - c_2 M_i \left( B_i^{eq} - k_i(p) \frac{W_i - \mathcal{D}_i(p)}{M_i} \right)^2$$

Then using

$$\frac{\partial \mathcal{P}_i(p)}{\partial p_i} = 0$$
Recall that the demand function can be expressed as $D_i(p) = D_1(p_i) - D_2(p_i)$. The solution $p^*$, which is a Nash equilibrium, can be obtained by solving the above set of linear equations by using a numerical method when all the parameters in above equation are available. Then, given a vector of prices $p^*$ at the Nash equilibrium, the size of the shared spectrum can be obtained from the spectrum demand function $D_i(p^*)$.

SPECTRUM POOLING PRICING

For Spectrum pooling the pricing strategy proposed is Cooperative Pricing Model. In the cooperative pricing model, a primary service provider is aware of other primary service providers, and the primary service providers cooperate with each other.

If the spectrum available to the secondary service creates high utility, the demand is high. To quantify the spectrum demand, we consider the utility gained by the secondary users. We adopt the following commonly used quadratic utility function:

$$U(b) = \sum_{i=1}^{N} b_i k^{(s)}_i - \frac{1}{2} \left( \sum_{i=1}^{N} b_i^2 + 2\nu \sum_{i \neq j} b_i b_j \right) - \sum_{i=1}^{N} p_i b_i$$

where $b$ is a vector of shared spectrum sizes from all the primary services, i.e., $b_1 ... b_i ... b_N$, $p_i$ is the price offered by primary service $i$ (N is the total number of primary services, i.e., spectrum sellers). Note that $k(i)^{(s)}$ denotes the spectral efficiency of wireless transmission by the secondary users using frequency spectrum $F_i$ owned by primary service $i$. This utility function takes the spectrum substitutability into account through the parameter $\nu$ ($0.0 \leq \nu \leq 1.0$). That is, if a secondary user uses a multi-interface network adaptor, it is able to switch among the frequency spectra freely depending on the offered price. This spectrum substitutability parameter $\nu$ is defined as follows.

When $\nu=0.0$, a secondary user cannot switch among the frequency spectra, while for $\nu=1.0$ the secondary user can switch among the operating frequency spectra freely.
The demand function for spectrum $F_i$ at the secondary service can be obtained using
\[
\frac{\partial \bar{y}(b)}{\partial b_i} = 0
\]
as follows:

\[
D_i(p) = \frac{(k_i^{(s)} - p_i)(\nu(N - 2) + 1) - \nu \sum_{i \neq j} (k_j^{(s)} - p_j)}{(1 - \nu)(\nu(N - 1) + 1)}
\]

where $p$ denotes a vector of prices offered by all primary services in the market (i.e., $p = p_1 \ldots p_i \ldots p_N$).

The demand function can be rewritten as follows:

\[
D_i(p) = D_1(p - i) - D_2 p_i,\text{where } (p - i) \text{ denotes the vector of prices of all primary services except service } i, D_1(p - i) \text{ and } D_2 \text{ are constants for given } p_j \text{ for } i \neq j \text{ which are given as follows:}
\]

\[
D_1(p - i) = \frac{k_i^{(s)}(\nu(N - 2) + 1) - \nu \sum_{i \neq j} (k_j^{(s)} - p_j)}{(1 - \nu)(\nu(N - 1) + 1)}
\]

\[
D_2 = \frac{(\nu(N - 2) + 1)}{(1 - \nu)(\nu(N - 1) + 1)}.
\]

**Solution of Co-operative Pricing Model**

The solution of this pricing model gives an optimal price for which the total profit of all primary services is maximized. Again, we assume that a primary service can observe the variation of spectrum demand from the secondary service. In addition, to achieve the highest total profit, primary services can exchange information on current profit among each other. This distributed cooperative pricing works as follows. The spectrum price is initialized to $p_i$ and then it is sent to the secondary service. The secondary service replies with the size of spectrum demand. Then, a primary service estimates marginal total profit by exchanging information with the rest of the primary services and uses this together with the spectrum demand from secondary service to compute spectrum price in the next iteration.

**PRICING MODEL**

For the cooperative pricing model, an formulated to obtain the optimal price which provides the highest total profit for all primary services. This optimization problem can be expressed as follows:
Maximize:  \[ \sum_{i=1}^{N} P_i(p) \]
Subject to:  
\( W_i \geq b_i \geq 0 \)
\( p_i \geq 0 \)

Where the total profit for all the primary services is given by \( \sum_{i=1}^{N} P_i(p) \)

\( W_i \geq b_i \geq 0 \) can be written as \( W_i \geq \mathcal{D}(p) \geq 0 \), the Lagrangian can be expressed as follows:

\[
\mathcal{L}(p) = \sum_{i=1}^{N} P_i(p) - \sum_{j=1}^{N} \lambda_j (-p_j) - \sum_{k=1}^{N} \mu_k (\mathcal{D}_k(p) - W_k) - \sum_{l=1}^{N} \sigma_l (-\mathcal{D}_l(p))
\]

Where \( \lambda_j, \mu_k, \sigma_l \) are Lagrange multipliers.

Using the Kuhn-Tucker conditions, we can obtain the vector of optimal prices \( P^* \) such that the total profit of all primary services are maximised.

**COSTS TO BE CONSIDERED**

1. SLA not fulfilled – forfeiting amount
2. Spectrum efficiency in accordance with number of users
3. Priority costing: more priority more charges
4. Spectrum rental
5. Trade-off between Larger sub-band size which enables faster processing, and smaller size which improves the fineness of the allocation.
SPECTRUM SHARING PRICING

For spectrum sharing we propose to follow the market equilibrium based pricing model. In the market-equilibrium pricing model, it is assumed that the primary service user is not aware of others. In an actual environment, this could be due to the lack of any centralized controller or information exchange among primary services. As a result, at the seller side, the primary service naively sets the price according to the spectrum demand of the secondary service. This price setting is based on the willingness of the primary service to sell spectrum which is generally determined by the supply function. For a given price, supply function indicates the size of radio spectrum to be shared by a primary service with the secondary service. At the buyer side, the willingness of a secondary service to buy spectrum is determined by the demand function. Again, for a given price, demand function determines the size of radio spectrum required by a secondary service. In this spectrum trading, market-equilibrium price denotes the price for which spectrum supplied by the primary service is equal to the spectrum demand from the secondary service. This market-equilibrium price ensures that there is no excess supply in the market and spectrum supply meets all spectrum demand.

Solution of Market-Equilibrium Pricing Model

For each primary service, the spectrum supply function can be derived based on a profit maximization problem. The solution of this optimization formulation is the optimal spectrum size $b_i^*$ to be shared with the secondary service for a given price $p_i$. Based on revenue (i.e., $R_i$ and $R_i(p)$) and cost (i.e., $C_i$) profit $P_i$ of a particular primary service $i$ owning spectrum $f_i$ can be expressed as follows:

$$P_i = p_i b_i + c_i M_i - c_s M_i \left( b_i^{req} - k_i(p) \frac{W_i - b_i}{M_i} \right)^2$$

To obtain the optimal spectrum size to be shared, we differentiate the profit function with respect to $b_i$ (when $p_i$ is given) as follows:

$$\frac{\partial P_i}{\partial b_i} = 0 = -p_i + \frac{2c_i M_i \left( b_i^{req} - k_i(p) \frac{W_i - b_i}{M_i} \right) \left( k_i(p) \right)}{M_i}$$

Spectrum supply is given by the optimal value of $b_i^*$ which is a function of price $p_i$. The supply function can be expressed as follows:

$$S_i(p_i) = W_i - \frac{M_i}{k_i(p)} \left( b_i^{req} - \frac{p_i}{2c_i k_i(p)} \right)$$

The market-equilibrium (i.e., solution) is defined as the price $p_i^*$ at which spectrum supply equals spectrum demand, i.e., $S_i(p_i^*) = D_i(p)$, $\forall i$ where the vector $P^* = [\ldots p_i^* \ldots]^T$ denotes the market-equilibrium prices for all primary services.
CONCLUSION

This paper covers entire end to end modelling for all possible methods of spectrum transactions and gives the pricing model based on which the operators can go ahead with the pricing of spectrum in each of the case. This was our sincere attempt to help the profit starving operators to utilize their biggest asset that is the spectrum at right price. We believe that identifying most suitable business model depends on the economic viability and commercial feasibility therefore firstly we have helped the operators to identify which model they should go for in the form of pricing matrix and then we have described how to go about the pricing. As we find a very high probability of it getting implemented because Nation Telecom Policy is going to define the Exit path for the operators and it has also proposed Spectrum sharing, trading and pooling.

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